The Lambert *W* Function

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2 Applications

Because W is a so-called implicitly elementary function, meaning it is defined as an implicit solution of an equation containing only elementary functions, it

the iteration converges if $|\ |< 1$, and also if $= e^{i}$ for $\ $ equal to some $\ $ multiple of $\ $, say $m\ /k$. Regions where the iteration converges to a k-cycle may touch the unit circle at those points.

4 Retrospective

The Lambert W function crept into the mathematics literature unobtrusively, and it now seems natural there. There is even a matrix version of it, although the solution of the matrix equation $Se^{S} = A$ is not always $W(A)$.

Hindsight can, as it so often does, identify the presence of W in writings by Euler, Poisson, and Wright and in many applications. Its implementation in Maple in the early 1980s was a key step in its eventual popularity.

Indeed, its recognition and naming supports Alfred North Whitehead's opinion that:

> By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems.

Further Reading

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