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The asymptotic approach of two parallel spheres has been studied by Batchelor (1969), Hansford (1970) and Jeffrey (1982) with the aim of calculating asymptotically the forces exerted by the spheres on the fluid. These forces are not the only quantities of interest, however, because the analysis of the properties of suspensions of spheres requires also the stresses of the spheres, defined by

$$- \int_A \mathbf{x}' \cdot \boldsymbol{\sigma} \cdot \mathbf{n} \, dA, \quad (1.1)$$

where the vector \mathbf{x}' is drawn from the centre of the sphere. Various authors have defined resistance

of ψ . This is not so for the stresslet, however. For its calculation we require the pressure

$$p = -\frac{1}{2} \nabla^2 \psi \quad (2.10)$$

and the Reynolds equation for the pressure is simpler in the

non-axisymmetric case is that

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$$\nabla^2 p = \nabla^2 \psi \quad (2.19)$$

$\nabla U = \Omega_{\infty} \mathbf{e}_z$. One can reduce the streamlets to expressions containing the circular functions

$$U = \frac{1}{2} \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(\frac{1}{2} \mu^2 + \frac{1}{2} \mu^4 + \frac{1}{2} \mu^6 + \dots \right) + \frac{1}{2} (1 + \mu^2) \left(\frac{1}{2} \mu^2 - \frac{1}{2} \mu^4 + \frac{1}{2} \mu^6 - \dots \right) \right] \quad (1)$$

$$W = \frac{1}{2} \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(\frac{1}{2} \mu^2 - \frac{1}{2} \mu^4 + \frac{1}{2} \mu^6 - \dots \right) + \frac{1}{2} (1 + \mu^2) \left(\frac{1}{2} \mu^2 + \frac{1}{2} \mu^4 - \frac{1}{2} \mu^6 + \dots \right) \right] \quad (2)$$

$$\psi = \frac{1}{2} \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(\frac{1}{2} \mu^2 + \frac{1}{2} \mu^4 + \frac{1}{2} \mu^6 + \dots \right) - \frac{1}{2} (1 + \mu^2) \left(\frac{1}{2} \mu^2 - \frac{1}{2} \mu^4 + \frac{1}{2} \mu^6 - \dots \right) \right] \quad (3)$$

$$\chi = \frac{1}{2} \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(\frac{1}{2} \mu^2 - \frac{1}{2} \mu^4 + \frac{1}{2} \mu^6 - \dots \right) - \frac{1}{2} (1 + \mu^2) \left(\frac{1}{2} \mu^2 + \frac{1}{2} \mu^4 - \frac{1}{2} \mu^6 + \dots \right) \right] \quad (4)$$

$$\frac{\partial U}{\partial \mu} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(\mu + \mu^3 + \mu^5 + \dots \right) + \frac{1}{2} (1 + \mu^2) \left(\mu - \mu^3 + \mu^5 - \dots \right) \right] \quad (5)$$

$$\frac{\partial W}{\partial \mu} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(\mu - \mu^3 + \mu^5 - \dots \right) + \frac{1}{2} (1 + \mu^2) \left(\mu + \mu^3 - \mu^5 + \dots \right) \right] \quad (6)$$

$$\frac{\partial \psi}{\partial \mu} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(\mu + \mu^3 + \mu^5 + \dots \right) - \frac{1}{2} (1 + \mu^2) \left(\mu - \mu^3 + \mu^5 - \dots \right) \right] \quad (7)$$

$$\frac{\partial \chi}{\partial \mu} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(\mu - \mu^3 + \mu^5 - \dots \right) - \frac{1}{2} (1 + \mu^2) \left(\mu + \mu^3 - \mu^5 + \dots \right) \right] \quad (8)$$

$$\frac{\partial^2 U}{\partial \mu^2} = \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(1 + 3\mu^2 + 5\mu^4 + \dots \right) + \frac{1}{2} (1 + \mu^2) \left(1 - 3\mu^2 + 5\mu^4 - \dots \right) \right] \quad (9)$$

$$\frac{\partial^2 W}{\partial \mu^2} = \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(1 - 3\mu^2 + 5\mu^4 - \dots \right) + \frac{1}{2} (1 + \mu^2) \left(1 + 3\mu^2 - 5\mu^4 + \dots \right) \right] \quad (10)$$

$$\frac{\partial^2 \psi}{\partial \mu^2} = \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(1 + 3\mu^2 + 5\mu^4 + \dots \right) - \frac{1}{2} (1 + \mu^2) \left(1 - 3\mu^2 + 5\mu^4 - \dots \right) \right] \quad (11)$$

$$\frac{\partial^2 \chi}{\partial \mu^2} = \Omega_{\infty} \left[\frac{1}{2} (1 - \mu^2) \left(1 - 3\mu^2 + 5\mu^4 - \dots \right) - \frac{1}{2} (1 + \mu^2) \left(1 + 3\mu^2 - 5\mu^4 + \dots \right) \right] \quad (12)$$

$$\frac{\partial^3 U}{\partial \mu^3} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) + \frac{1}{2} (1 + \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) \right] \quad (13)$$

$$\frac{\partial^3 W}{\partial \mu^3} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) + \frac{1}{2} (1 + \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) \right] \quad (14)$$

$$\frac{\partial^3 \psi}{\partial \mu^3} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) - \frac{1}{2} (1 + \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) \right] \quad (15)$$

$$\frac{\partial^3 \chi}{\partial \mu^3} = \Omega_{\infty} \mu \left[\frac{1}{2} (1 - \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) - \frac{1}{2} (1 + \mu^2) \left(-2\mu + 6\mu^3 - 10\mu^5 + \dots \right) \right] \quad (16)$$

REFERENCES

that the sphere of radius a approaches the other which is at rest at \dots
The scale velocity \mathcal{V} is then U and the boundary conditions to be applied on
e:

We shall suppose
velocity U .
sphere a an

$$U = 0 \quad \text{and} \quad W = -1 \quad \text{on} \quad Z = H_1 + \varepsilon \frac{1}{2} R^2,$$

The

$$H = 0 \quad W = R_0 + \frac{1}{2} \epsilon^2 R_2 + \frac{1}{6} \epsilon^4 R_4 + \dots + \epsilon^2 K^* K$$

Since

$$R_0(H_1) + \epsilon U_1(K, H_1) + \epsilon^2 K^* \partial U_1 / \partial Z|_{Z=H_1},$$

$$U(R_0 Z - H_1 + \frac{1}{2} \epsilon^2 K^* K) = U_1(K,$$

we can equate powers of ϵ to find





conditions into

stretched coordinates, we obtain:

$$W = -1 + \alpha(1-z) \quad \text{and} \quad U = -\frac{1}{2}e^{1/2}R.$$

Therefore, the boundary conditions on the deforming sphere become

$$W(R, H) = 1 \quad \text{and} \quad W(R, H) = R^{-1/2}H^{3/2}.$$

are the same as in the previous

section. The method of solution is also the same and

again the pressure, where R is large

goes as $O(R^{-2}) + eO(R^{-2})$. Since this flow is not

is not been solved before, we have

the stresslet to calculate. However,

the force calculation does not give

any new results, because the reciprocal of the

shows that the forces are proportional to

the resistance matrix is symmetric

Similar results to Y_{10}^* are the same as those on Y_{10}^* , but we noticed that an overall bias is present in the Y_{10}^* results. This is shown in Figure 10.



Therefore we proceed to calculate the $O(\epsilon^2)$ constants, we write a general ansatz $X_{ij}^G(\epsilon, \lambda) = X_{ij}^G(\epsilon, \lambda) + \epsilon^2 X_{ij}^G(\epsilon, \lambda)$, in the special case $\lambda = 1$ using a global ansatz we find in the following that the functions $X_{ij}^G(\epsilon, \lambda)$ and $X_{ij}^G(\epsilon, \lambda)$ contain singular terms in ϵ^{-1} , ϵ^{-2} and ϵ^{-3} and we expect in the new functions studied here will be the same. We also note from (2.11)–(2.14) that the coefficients of the ϵ^{-1} and ϵ^{-2} terms of X_{ij}^G and X_{ij}^G are equal. If we suppose that this will be true for the ϵ^{-3} term as well we can write $X_{ij}^G(\epsilon, \lambda) = X_{ij}^G(\epsilon, \lambda) + \epsilon^2 X_{ij}^G(\epsilon, \lambda)$.

The constants $O(\epsilon^1)$ and X_{ij}^G will be obtained from (2.11)–(2.14) as

$$(4.9) \quad X_{11}^G = \frac{31}{8\epsilon} - \frac{27}{80}, \quad \text{with } \mathcal{F}^G(\epsilon, \lambda) = \frac{117}{560} \epsilon \ln \epsilon^{3/2} + \lambda \epsilon^2$$

The same considerations and assumptions lead to

$$X_{11}^M = \frac{2}{5} \left(1 + \frac{1}{2} \epsilon \right) X_{11}^G$$

n	without g_{∞}	with g_{∞}	without g_{∞}	with g_{∞}
100	0.716327	0.71631	-0.14194	-0.14493
200	0.71636	0.71636	-0.14424	-0.14574