Th a c ana n r a n n n ar r c rr nc r a n

o ert M. Corless, David J. Jeffrey and Fei ang

CCA and The Department of Applied Mathematics
The niversity of estern ntario
London, ntario, Canada

ABSTRACT

e study discrete dynamical systems, or recurrence relations, of the general form

$$y_{+1} = y \ \varphi(y$$

the f ed point is attracti e, and $|F'( ) > \ |$, for hich it is repelling, are more common but easier to understand

e associate ith the discrete system an interpolating continuous system $Y(t)$, such that $Y(n) = y$ The asymptotic beha iour of y can then be in estigated through $Y(t)$ A theorem of abelle states that the corresponding continuous system is

$$Y'(t) = G(Y(t)) \,, \qquad ( )$$

here G is termed the generator, and is gi en by

$$G(y) = \frac{\alpha_1 y^2}{+\beta_1 y + \beta_2 y^2 + \cdots} \,. \qquad ( )$$

The β are related to the α by the recurrence relation

$$\beta_0 = \ , \qquad ( )$$

5

$$\beta = \frac{}{n\alpha_1}\sum_{i=1}\left[P_{i+1}( ) - \frac{(n+ \ )P_{i+1}(n+ \ -i)}{n+ \ -i}\right]\beta_{-i}. \qquad ( )$$

here

ore such identities can be found in
The e pansion of logarithm is thus

$$\ln Y \sim \ln\left(Z + \sum_{\geq 2} c \; Z\right) \qquad (\quad)$$

$$\sim \ln Z + \ln\left(\; + \sum_{\geq 2} c \; Z^{\;-1}\right) - \; \pi i \mathcal{K}_0 \qquad (\quad)$$

$$\sim \ln Z + \sum_{\geq 1} d \; Z \; - \; \pi i \mathcal{K}_0 \; , \qquad (\quad)$$

here $\mathcal{K}_0 = \mathcal{K}(\ln Z + \ln(\; + \sum_{\geq 2} c \; Z^{\;-1}))$ The d are easily computed functions of the c by ell-kno n algorithms for the computation of the logarithm of a series [5] imilarly, by reciprocation of a series, e can obtain

$$\frac{\ }{Y} \sim \frac{\ }{Z}\left(\; + \sum_{\geq 1} e \; Z\right.$$

E[redacted]

e i nore the hi her order term and su stitute f $(z) =$ Z into ( ) and use the aple command *as mpt* to calculate the asymptotic series of urprisin ly, e et

$$(z) = -\frac{}{C(z)} + O(\frac{}{n}) \qquad (5)$$

That means

$$(z) = \lim_{\to\infty} (z) = -\frac{}{C(z)} \qquad ( )$$

This i es a ne approach for computin (z)

e use the am ert W function as an e ample to compare this ne approach to the strai htfor ard approach e call the strai htfor ard approach method and our ne approach method e compute (.) y each method

ethod

- n = , (.) = .
- n = , (.) = . 5 5

ethod

- n = 5, (.) = . 5
- n = , (.) = . 5
- n = 5 , (.) = . 5

As e can see, method has the ad anta e of con er in faster than method

7. CONCLUDING REMARKS

t has lon een no n in the analysis of al orithms community that usin the am ert function (also called sometimes the me a function, prior to its name ein standardi ed) as useful in economi in some asymptotic analyses This present paper sho s an e ample class of pro lems for hich the economi e 5 A (nal a)- 5 (ad n (ad n A)- (recono (y h)- 5 (ttd)- h P i(pro lems)- (rec