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Indefinite integration as term rewriting: integrals containing tangent

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Поступила в р дакцию

W d scrib th d v lopm nt of a t rm-r writing syst m for ind finit int gration; it is also call d a rul -bas d valuation syst m. Th d v lopm nt is s parat d into modul s, and w d scrib th modul for a wid class of int grands containing th tang nt function.

1. Introduction

Programming styles in computer-algebra systems are frequently described as either term-rewriting based, or computationally based. For example, Mathematica is widely recognized as a rewrite language [1], whereas Maple is rarely described this way. The distinction is mostly one of emphasis, since all available systems include elements of both styles of programming. The dichotomy can be seen more specifically in programming to evaluate indefinite integrals, also called primitives or anti-derivatives. For symbolic integration, some of the best-known approaches are computationally based. Thus the Risch algorithm [8] and the Rothstein-Trager-Lazard-Rioboo algorithm [9, 10,] are both computational algorithms. These algorithms and others like them are not universally applicable, however, and for many integrals rule-based rewriting is needed and has advantages, some of which we discuss below.

Doubts have been expressed about the viability of large-scale term rewriting [2]; the present scheme is more algorithmic and deterministic than earlier term rewriting schemes, and we prefer the description rule-based scheme for the integration scheme presented here. A particular success of rule-based schemes is the popularity of software that can display the steps of a calculation. This is variously called 'display step' or 'single step'. Examples of software offering this include WolframAlpha and Derive, and many calculus tutorial programs.

The rule-based integration scheme that is considered here [7, 6] consists of a public-domain repository of transformation rules for indefinite integration, together with utility files that allow it to be utilized by various computer-algebra systems. The repository is *not* a table of integrals; it is a compact set of rules, much smaller than a table covering the same domain. Also simple correctness is not the only aim of the development. The *quality* of integral expressions is judged by a number of criteria, which are used to decide whether an integration rule should be accepted. Assume that an integrand $f(x)$ has a proposed primitive $F(x)$. Selection is based on the following criteria, which are discussed further in the next section.

- Correctness: we require $F'(x) = f(x)$.
- Simplicity: we seek the simplest form for an integral. We adopt a pragmatic approach and aim for the shortest expression length.
- Continuity: we aim to ensure that all of the expressions for integrals are continuous on domains of maximum extent [4].
- sthetics: we employ a number of principles to select for mathematical beauty where possible.
- Utility: the rules should facilitate the aforementioned 'show-step' application. See Sec. 2. .
- Efficiency: the path to a result should be as direct as possible, and the set of rules should be compact.

A more detailed description of the repository is given below.

2. Discussion of selection criteria

The following example allows us to discuss several aspects of our overall aim. We compare possible expressions for an integral.

$$\int \sqrt{-2\tan x}\, dx = -- \ln\left(2 - 2\tan x - 2\sqrt{-2\tan x}\right) - \arctan\left(\right.$$

Consider the following example, which comes from MAPLE 16.

$$\int \sqrt{2\tan x}\; dx = \frac{\sqrt{\tan x}\cos x \arccos(\cos x - \sin x)}{\sqrt{\cos x \sin x}} - \ln\left(\cos x + \sqrt{2}\sqrt{\tan x}\cos x + \sin x\right) : \quad (10)$$

In contrast to (1), also from Maple, this form introduces a jumble of functions not seen in the integrand. Also the reduce integrands to forms for which the evaluation is known. The final rule is often called a termination rule. We have not listed all the rules here, because with the application conditions and termination rules, there are

7. Appendix

We list the recurrence relations used in the integration scheme [11]. To save space and to show the structure more clearly, we use the abbreviations

$$T = \tan(c + dx) \, ; \qquad T = a + b \tan(c + dx) \, ;$$
$$T (A, B, C) = A + B \tan(c + dx) + C \tan (c + dx) \, .$$

Then the recurrences, valid for all $A, B, C \in \mathbb{C}$, are

$$d(m + 1)$$

$$ad(m+n)\int T^m T^n T\ (A;B;0)\,dx = \qquad (27)$$
$$aBT^m T^n + d\int T^{m-}\ T^n \hat{T}\ (\hat{A};\hat{B};0)\,dx\ ;$$
$$\hat{A} = -aBm\ ;\quad \hat{B} = Aam + (Aa - Bb)n\ :$$

$$ad(m+1)\int T^m T^n T\ (A;B;0)\,dx = \qquad (28)$$
$$aAT^m\quad T^n + d\int T^m\quad T^n \hat{T}\ (\hat{A};\hat{B};0)\,dx\ ;$$
$$\hat{A} = Abn - Ba(m+1)\ ;\quad \hat{B} = Aa(m+n+1)\ :$$

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