Symbolic-Numeric Computation
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A Symbolic-Numeric Approach to an Electric Field Problem

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Abstract. A combination of symbolic and numerical methods is used to extend the reach of the purely symbolic methods of physics. One particular physics problem is solved in detail, namely, a computation of the electric potential in the space between a sphere and a containing cylinder. The potential is represented as an infinite sum of multipoles, whose coefficients satisfy an infinite system of linear equations. The system is solved first symbod[illegible]e radius to cylinder radius. Purely symbolic methods, however, cannot complete the solution for two reasons. First, the coefficients in the series expansion can only be found numerically, and, second, the convergence rate of the series is too slow. The combination of symbolic and numerical methods allows the singular nature of an important special case to be identified.

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Keywords. Laplace equation, series solution, asymptotic solution, convergence, numerica [illegible]

_Bg ! T(T (9?Y9e 9^ 1KT

51K5aK1^HOM c1Z Y9Z^YH5^98 ^O Q1Y^H5K9Z O> ZQF9YH51K ZF1Q9T O H^F ^F9Z9 O2Z9Yb1^HOMZ HM LHM87 ZaQQ

.eL2OKH5G+aL9YH5 QQYO15F _BP

e 1QQKeHME ^F9Z9 H89M^H^H9Z c9 898a59

$$- \quad < \sum_{n=1}^{\infty} \frac{n}{`2} \frac{0^{2n+1}}{9^{2n}} \%_{2n-1} R5OZ<S$$

$$U \sum_{m,n=1}^{\infty} \frac{` \; {}_n R - PS^{n+m}}{7R`2S{=}R`1 - PS{=}} 0^{2n+1} 9^{2m-1} \%_{2m-1} R5OZ<S \int_0^{\infty} ;^{2(n+m-1)} \frac{"_1 R;S}{!_1 R;S} +; \; 5$$

$YOL ^F9 20aM81

HM89Q9M89M^ O> OT 'M OY89Y ^O ZQ998 aQ ^F9 5O L Qa^1^HOM 1M8 ^O H L QYOb9 H^Z Y9KHG 12HKH^e7 c9 51M HM^9EY1^9 ^F9 8O L HM1M^ 5OM^YH2a^HOM ^O ^F9 HM^9EY1K Ze L 2OKH51KKeT / FaZ7 c9 cYH^9

$$\int_0^{\infty} \frac{;^{2(n+m-1)}}{!_1^2 R;S} +; < \quad 7R`2 - PS$$

/ F9 ZaY>159 O> ^F9 ZQF9Y9 HM ^F9 M9c 5OOY8HM1^9Z F1Z ^F9 9dQ1MZHOM

$$\& < P U \frac{P}{`}\, '^{2} U . \left( \frac{'^{2}}{`} U \frac{'^{4}}{:} \right) U \$ R.^{2} S5$$

O9 cHKK 1QQKe ^F9 2OaM81Ye 5OM8H^HOM OM ^FHZ 1QQYOdH L 1^HOM Y1^F9Y ^F1M OM ^F9 9d15^ ZaY>159? ZHM59 ^F9 9d15^ 9dQY9ZZHOM 5OM^1HMZ ZVa1Y9 YOO^ZT

/ F9 Z51K98) 1QK159XZ 9Va1^HOM HMZH89 ^F9 E1Q HZ EHb9M 2e

$$\frac{4^{2} -}{4\&^{2}} U . \left(\frac{4^{2} -}{4\, '^{2}} - \frac{P}{P - .\&} \cdot \frac{4 -}{4\&} \right) < g \qquad R`TPPS$$

1M8 ^F9 2OaM81Ye 5OM8H^HOM OM ^F9 5eKHM89Y 2e

$$\frac{4 -}{4\&} < g \qquad OM \quad \& < g5$$

'M OY89Y ^O 898a59 ^F9 2OaM81Ye 5OM8H^HOM OM ^F9 O2Z^15K9 c9 MO^H59 ^F1^

$$ZHM < < \frac{P - .\&}{P - .}* \qquad 5OZ < < \frac{\sqrt{.}\,'}{P - .} \qquad OM \quad 9 < P - .$$

1M8 89YHb9 ^F9 Z51K98 2OaM81Ye 5OM8H^HOM OM ^F9 ZQF9Y9

4 - 9

.eL2OKH5G+aL9YH5 QQYO15F _B\

TABLE 1. 5O L Q1YHZOM O> ^F9 5O L Qa^98 1M8 QY98H5^98 5O9@G 5H9M^Z HM ^F9 Z9YH9Z >OY ^F9 HM5Y91Z98 Y9ZHZ^1M59 O> 1 5OM8a5^HME 5eKHM89Y 8a9 ^O ^F9 QY9Z9M59 O> 1 ZQF9YH51K 2a22K9T

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